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Further Studies in Aesthetic Field Theory VI: The Lorentz Group

M. MURASKIN and BEATRICE RING

University of North Dakota, Grand Forks, North Dakota

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Abstract

In studying $\Gamma^i_{jk;l} = 0, g_{ij;k} = 0$ field theory we require that the underlying structure $(\Gamma^\alpha_{\beta\gamma}, g_{\alpha\beta})$ be invariant under $L(4)$, the four-dimensional Lorentz group. This can be accommodated into the theory by increasing the dimension to five. In our computer studies we still found a turnabout point for g_{44} on running down the x-axis, suggesting that this group may be consistent with a bounded particle. However, with still longer runs down the x-axis, there was some indication that a singularity may be developing.

1. *Introduction*

We have been able to obtain particle-like behavior by combining ‘aesthetic’ data at the origin with ‘aesthetic’ field equations (Muraskin, 1973a, 1973b; Muraskin & Ring, 1973, 1974a). The α, β variables in those papers were required to be invariant under space-time groups associated with Newtonian physics. It is a reasonable question to ask what sort of results we would get if instead we require $\Gamma^\alpha_{\beta\gamma}, g_{\alpha\beta}$ to be invariant under the Lorentz group. We shall find that such invariance is consistent with $\Gamma^i_{jk;l} = 0, g_{ij;k} = 0$ in five-dimensional space. In the computer runs we have made, we found a bound on g_{44}. However, still longer runs showed some indication that singularity may be developing.

2. *Lorentz Invariant Data*

We consider

$$\Gamma_{\alpha\beta\gamma} = \phi_\gamma g_{\alpha\beta} + \psi_\alpha g_{\beta\gamma} + \theta_\beta g_{\alpha\gamma} + \mu_\alpha \lambda_\beta \chi_\gamma \tag{2.1}$$

We choose

$$g_{\alpha\beta} = (1, 1, 1, -1, 1) \tag{2.2}$$

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(we get no significant differences on the computer if we choose $g_{\alpha\beta} = (1, 1, 1, -1, 0)$) and

$$\begin{aligned} \phi_1 &= \phi_2 = \phi_3 = \phi_4 = 0 \\ \mu_\alpha &= b\phi_\alpha \\ \lambda_\alpha &= c\phi_\alpha \\ \chi_\alpha &= d\phi_\alpha \\ \psi_\alpha &= \phi_\alpha = \theta_\alpha \\ \mu_5\lambda_5\chi_5 &= -2\phi_5 \end{aligned} \tag{2.3}$$

Then, $g_{\alpha\beta}$, $\Gamma_{\alpha\beta\gamma}$ obey the $R^i_{jkl} \neq 0$ integrability equations. We note $\Gamma_{\alpha\beta\gamma}$ is, thus, symmetric in all indices. We could have taken, say, $\psi_\alpha \neq \phi_\alpha$ and still have found that integrability would be satisfied. However, in view of the fact that $\Gamma_{\alpha\beta\gamma}$ is already not completely general, since it does not have any completely antisymmetric part, it seemed more reasonable to further restrict ourselves to a completely symmetric $\Gamma_{\alpha\beta\gamma}$. We note in Muraskin (1974) a completely symmetric $\Gamma_{\alpha\beta\gamma}$ was studied. Also in Muraskin (1974) we did not find any obvious usefulness for the antisymmetric part in connection with $O(3)$-type theories.

We see that (2.1) and (2.2) are invariant under four-dimensional Lorentz transformation. In order to get a Lorentz invariant $\Gamma_{\alpha\beta\gamma}$ we were led to five dimensions as we do not see how we can get invariance in four dimensions.

We have mentioned above that there is no non-vanishing completely antisymmetric contribution associated with (2.1). Such a term would need to have the structure

$$B^\lambda C^\chi \epsilon_{\lambda\chi\alpha\beta\gamma} \tag{2.4}$$

But to construct a form invariant under $L(4)$ would require B^λ to be zero if $\lambda \neq 5$ and C^χ to be zero for $\chi \neq 5$. That is, we construct a $L(4)$ invariant $\Gamma_{\alpha\beta\gamma}$ from a vector of structure $(0, 0, 0, 0, a)$ and a $g_{\alpha\beta}$ of structure $(1, 1, 1, -1, b)$. Now, if $\lambda = \chi = 5$, we see from the properties of the antisymmetric symbol that (2.4) is equal to zero.

We chose in our computer runs the following values for the non-vanishing $\Gamma^\alpha_{\beta\gamma}$

$$\begin{aligned} \Gamma^1_{15} &= \Gamma^2_{25} = \Gamma^3_{35} = \Gamma^4_{45} = \Gamma^5_{55} = 1 \\ \Gamma^1_{51} &= \Gamma^2_{52} = \Gamma^3_{53} = \Gamma^4_{54} = 1 \\ \Gamma^5_{11} &= \Gamma^5_{22} = \Gamma^5_{33} = -\Gamma^5_{44} = 1 \end{aligned} \tag{2.5}$$

We calculated $e^4{}_1$, $e^4{}_2$, $e^4{}_3$ in the manner of Muraskin (1971) and obtained a maximum in g_{44} at the origin using

$$\begin{array}{lllll} e^1{}_1 = 0{\cdot}9 & e^1{}_2 = -0{\cdot}13 & e^1{}_3 = -0{\cdot}187 & e^1{}_4 = -0{\cdot}034 & e^1{}_5 = 1{\cdot}2 \\ e^2{}_1 = 0{\cdot}21 & e^2{}_2 = 0{\cdot}5 & e^2{}_3 = -0{\cdot}24 & e^2{}_4 = 0{\cdot}017 & e^2{}_5 = 0{\cdot}082 \\ e^3{}_1 = -0{\cdot}17 & e^3{}_2 = -0{\cdot}26 & e^3{}_3 = 0{\cdot}65 & e^3{}_4 = -0{\cdot}042 & e^3{}_5 = 0{\cdot}29 \\ & & & e^4{}_4 = -0{\cdot}71 & e^4{}_5 = 0{\cdot}51 \\ e^5{}_1 = -0{\cdot}05 & e^5{}_2 = -0{\cdot}04 & e^5{}_3 = -0{\cdot}038 & e^5{}_4 = 0{\cdot}22 & e^5{}_5 = 1{\cdot}01 \end{array} \tag{2.6}$$

The calculation yielded

$$e^4{}_1 = -0{\cdot}0444 \qquad e^4{}_2 = -0{\cdot}0915 \qquad e^4{}_3 = -0{\cdot}0198$$

These numbers have been rounded off here.

Running the computer down the x-axis we found the following values for g_{44} and g_{55}.

x	g_{44}	g_{55}
0	−0·45	2·29
0·5	−0·55	7·46
1·0	−1·17	16·0
1·05	−0·64	515
1·06	−0·18	686
1·075	1·37	1,155
1·093	8·12	2,801
1·105	28·50	7,085
1·1115	70·88	15,314
1·114	110·91	22,820

We see that g_{00} stopped decreasing at about $x = 1{\cdot}0$ but then began to increase quite dramatically in magnitude. g_{55} was already so large at $x = 1{\cdot}114$ that running on the computer was discouraged. The computer cannot tell us that a singularity is present. However, the suggestion that this may be happening is there. We did not notice anything particularly encouraging about the behavior of Γ^i_{jk} during the run. We graphed 12 of the 125 components of Γ^i_{jk} during the run and found a net total of one turnabout point (in Γ^1_{11}). It may well be that $L(4)$ symmetry can lead to a bounded particle but with a singular structure outside the particle. No amount of running on the computer can prove this contention but it seems like a reasonable tentative supposition.

Thus we feel that $L(4)$ invariant data gives no indication, at present, of leading to an improvement over previously obtained results.

3. *Other Five-Dimensional Runs*

We took the $O(3)$ invariant $R^i_{jkl} \neq 0$ data appearing in Muraskin (1973b) and required that $\Gamma^\alpha_{\beta\gamma}$ be zero when α, β or γ took on the value 5. Thus, we may say that we have a four-dimensional substructure in a five-dimensional space. The results were then compared with the four-dimensional version of the data. We found that when $e^5{}_i$ and $e^\alpha{}_5$ were small, the results for Γ^i_{jk}, i, j, k = 1–4, differed slightly at the origin. After a long run down an axis we found that the five-dimensional and four-dimensional results approached one another for Γ^i_{jk}, i, j, k = 1–4. This was the same kind of results we had obtained in our eight-dimensional studies (Muraskin & Ring, 1974b). Thus, the higher dimensions do not lead to any improvement in this case. We also considered the same five-dimensional data above, but with $\Gamma^5_{55} = 1$ instead of zero. We found again that the four-dimensional and five-dimensional results approached one another.

We also tried the four-dimensional data appearing in Muraskin & Ring (1974b), having all invariants zero, and extended the data to five-dimensional space as above. Here the four-dimensional and five-dimensional data did not approach one another. We have (Muraskin & Ring, 1974b) pointed out that the four-dimensional data has singular structure. We found that the five-dimensional run did not appear to show any new effects so far as we could tell. Our tentative contention is that the five-dimensional run also has a singular structure. Note, we drew a similar contention when the same four-dimensional data was extended to eight-dimensions in Muraskin & Ring (1974b).

4. *Summary*

In attempting to improve our previous results, it is necessary to explore as many reasonable alternatives as we can think of. There is some indication from the computer that requiring $\Gamma^\alpha_{\beta\gamma}, g_{\alpha\beta}$ be invariant under the Lorentz group $L(4)$ may well lead to a singular structure. We were able to obtain invariance under $L(4)$ by increasing the dimensions to five. Other five-dimensional runs that we made did not appear to differ from the type of results we had in our eight-dimensional studies. Thus, we cannot say that our results are an improvement upon those of the past.

References

Muraskin, M. (1971). *International Journal of Theoretical Physics*, Vol. 4, No. 1, p. 49.
Muraskin, M. (1973a). *International Journal of Theoretical Physics*, Vol. 7, No. 3, p. 213.
Muraskin, M. (1973b). *International Journal of Theoretical Physics*, Vol. 8, No. 2, p. 93.
Muraskin, M. and Ring, Beatrice (1973). *International Journal of Theoretical Physics*, Vol. 8, No. 2, p. 85.
Muraskin, M. (1974). *International Journal of Theoretical Physics*, Vol. 9, No. 6, p. 405.
Muraskin, M. and Ring, Beatrice (1974a). *Foundations of Physics*, 4, 395.
Muraskin, M. and Ring, B. (1974b). *International Journal of Theoretical Physics*, Vol. 11, No. 2, p. 93.